Pearson Edexcel

Mark Scheme (Results)

Summer 2024

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1) Paper 01R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - cso – correct solution only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - awrt – answer which rounds to
    - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers may score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question: eg. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)
  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used.
  Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2 + bx + c) = (x + p)(x + q)$,where $|pq| = |c|$ leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \ldots$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x = \ldots$

3. Completing the square:

$x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$ leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n \rightarrow x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values **or**, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks". General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is given.

Question	Scheme	Marks
1	$\sqrt{50}x - \sqrt{18} > 6x + 5 \Rightarrow \sqrt{50}x - 6x > 5 + \sqrt{18} \Rightarrow x > \dfrac{5+\sqrt{18}}{\sqrt{50}-6}$	M1
	$\dfrac{5+\sqrt{18}}{\sqrt{50}-6} \times \dfrac{\sqrt{50}+6}{\sqrt{50}+6} = \dfrac{5\sqrt{50}+30+\sqrt{18}\sqrt{50}+6\sqrt{18}}{14} = \dfrac{43\sqrt{2}+60}{14}$	M1dM1
	$x > \dfrac{43\sqrt{2}+60}{14}$	A1 [4]
		Total 4 marks

Mark	Notes
M1	Collects like terms and obtains a value for x in the minimally acceptable form: $x > \dfrac{a+\sqrt{18}}{\sqrt{50}-6}$ o.e for example $x > \dfrac{a+3\sqrt{2}}{5\sqrt{2}-6}$ where a is an integer. Please watch out for reversed signs For example, accept $\dfrac{-a-\sqrt{18}}{-\sqrt{50}+6}$ Ignore the inequality and accept >, <, = for this mark
M1	For showing the intent to multiply the numerator and denominator by the conjugate of **their** $\left(\sqrt{50}+6 \text{ or } 5\sqrt{2}+6\right)$ but minimally of the form $\sqrt{A}+B$ or $C\sqrt{2}+D$ e.g., $\dfrac{5+\sqrt{18}}{\sqrt{50}-6} \times \dfrac{\sqrt{50}+6}{\sqrt{50}+6}$ scores this mark. That is all that is required for this mark. Ignore >, <, = for this mark
dM1	For explicitly multiplying out and simplifying their expression to obtain a value for x in the required form. Allow no more than one error in this simplification. However, it must be simplified as far as $\dfrac{A+\sqrt{B}}{C}$ or $\dfrac{P+Q\sqrt{R}}{T}$ where A, B, C, P, Q, R and T are integers Ignore >, <, = for this mark
A1	For obtaining the value of x as given with the correct inequality. For candidates who give an estimated value [8.629….] , please isw if you see the exact value first.

ALT 1 – Squares both sides [Ignore <, >, = for the first 3 marks]	
M1	Squares both sides without errors. $\left(\sqrt{50}x-\sqrt{18}\right)^2 > \left(6x+5\right)^2 \Rightarrow 50x^2-60x+18>36x^2+60x+25$
M1	Collects up their like terms and forms a 3TQ $\Rightarrow 14x^2-120x-7>0$
dM1	Solves their 3TQ by either valid method using the formula or completing the square. $x=\dfrac{60\pm 43\sqrt{2}}{14}$ oe eg., $x=\dfrac{60\pm\sqrt{3698}}{14}$ Accept $\pm$ for this mark We MUST see a method for the award of this mark. Do not award for roots appearing with no working. Ignore >, <, = for this mark
A1	For obtaining the value of x as given with the correct inequality. This has to be $x>\dfrac{43\sqrt{2}+60}{14}$ For candidates who give an estimated value [8.629….] , please isw if you see the exact value first.
ALT 2 – Collects up like terms and squares [Ignore <, >, = for the first 3 marks]	
M1	Squares both sides without error. $\left(\sqrt{50}x-6x\right)^2 > \left(5+\sqrt{18}\right)^2 \Rightarrow 50x^2-12\sqrt{50}x^2+36x^2>25+10\sqrt{18}+18$
M1	Collects up like terms and forms a **2TQ** $86x^2-12\sqrt{50}x^2-\left(43+10\sqrt{18}\right)>0 \Rightarrow \left[\left(86-60\sqrt{2}\right)x^2-\left(43+30\sqrt{18}\right)>0\right]$
dM1	Solves their 2TQ by either valid method using the formula or completing the square. $x=\dfrac{43\sqrt{2}\pm 60}{14}$ oe eg., $x=\dfrac{\sqrt{3698}\pm 60}{14}$ We MUST see a method for the award of this mark. Do not award for roots appearing with no working.
A1	For obtaining the value of x as given with the correct inequality. This has to be $x>\dfrac{43\sqrt{2}+60}{14}$ For candidates who give an estimated value [8.629….] , please isw if you see the exact value first.

Question	Scheme	Marks
2(a)	$(1+Ax)^n = 1+n(Ax)+\frac{n(n-1)(Ax)^2}{2}+\ldots$	B1
	$\Rightarrow nA = -\frac{1}{3} \qquad \frac{n(n-1)A^2}{2} = \frac{5}{36}$	M1
	$\Rightarrow A = -\frac{1}{3n}$	
	$\Rightarrow \frac{n(n-1)\left(-\frac{1}{3n}\right)^2}{2} = \frac{5}{36} \Rightarrow n = -\frac{2}{3} \Rightarrow A = \frac{1}{2}$	M1dM1A1A1 [6]
	ALT 1	
	$\Rightarrow nA = -\frac{1}{3} \qquad \frac{n(n-1)A^2}{2} = \frac{5}{36}$	[M1
	$\Rightarrow n = -\frac{1}{3A}$	
	$\Rightarrow \frac{-\frac{1}{3A}\left(-\frac{1}{3A}-1\right)(A)^2}{2} = \frac{5}{36} \Rightarrow A = \frac{1}{2} \Rightarrow n = -\frac{2}{3}$	M1dM1A1A1]
	ALT 2	
	$\Rightarrow nA = -\frac{1}{3} \Rightarrow \left[(nA)^2 = \frac{1}{9}\right]$	
	$\frac{n(n-1)A^2}{2!} = \frac{5}{36} \Rightarrow \left[\frac{(nA)^2 - A(An)}{2!} = \frac{5}{36}\right]$	[M1
	$\Rightarrow \frac{\frac{1}{9} - A\left(-\frac{1}{3}\right)}{2} = \frac{5}{36} \Rightarrow A = \frac{1}{2} \Rightarrow n = -\frac{2}{3}$	M1dM1A1A1]
(b)	Coefficient of $x^3 \Rightarrow \frac{\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(-\frac{8}{3}\right)\left(\frac{1}{2}\right)^3}{3!} = -\frac{5}{81}$	M1A1 [2]
		Total 8 marks

Part	Mark	Notes
(a)	B1	For the correct expansion of $(1+Ax)^n$ This can be implied by correct work in the term in x and x^2
	M1	Equates their second term to $-\frac{1}{3}$ and their third term to $\frac{5}{36}$ Allow presence of x and x^2, provided it is on **BOTH** sides. eg Allow $nAx = -\frac{1}{3}x$ and $\left[\frac{n(n-1)A^2}{2}\right]x^2 = \frac{5}{36}x^2$
	M1	Substitutes A into n to form an equation in n
	dM1	For solving **their** equation in n If they make errors in simplification to this step e.g., $\left(-\frac{1}{3n}\right)^2 \Rightarrow \left(\frac{1}{9n}\right)$ or $\left(-\frac{1}{3n}\right)^2 \Rightarrow -\left(\frac{1}{9n^2}\right)$ then this is M0 Their equation must be either a quadratic or linear if they cancel an n This is dependent on the previous M mark only.
	A1	For the correct value of n **or** A $n = -\frac{2}{3},\ A = \frac{1}{2}$ isw $n = 0$ seen. This is equivalent to cancelling n's
	A1	For the correct value of n **and** A
	ALT 1	**First two marks same as main method**
	M1	Substitutes n in terms of A into an equation in A $\frac{'-\frac{1}{3A}'\left('-\frac{1}{3A}'-1\right)(A)^2}{2} = \frac{5}{36}$ **Do not** allow a substitution of $n = -\frac{1}{3}$ for this mark
	dM1	For solving **their** equation in A to find a value for A Their equation must be either a quadratic or linear if they cancel an A This is dependent on the previous M mark only.
	A1	For the correct value of n **or** A $n = -\frac{2}{3},\ A = \frac{1}{2}$
	A1	For the correct value of n **and** A
	ALT 2	**First two marks same as main method**
	M1	Substitutes nA into $\frac{(nA)^2 - A(An)}{2!} = \frac{5}{36}$
	dM1	Solves their equation to find a value for A provided no errors introduced. This is dependent on the previous M mark only.
	A1	For the correct value of n **or** A $n = -\frac{2}{3},\ A = \frac{1}{2}$
	A1	For the correct value of n **and** A

(b)	M1	Uses the **correct form** for the fourth term of a binomial expansion with their A and their n. You may see this in terms of A and n in part (a) Accept as a minimum, the correct power of $\left(\frac{1}{2}\right)^3$ with the correct denominator i.e. 3! Allow the presence of x^3 for this mark.
	A1	For the correct value of $-\frac{5}{81}$ If you see the correct value following the correct values of A and n with no working, award M1A1

Question	Scheme	Marks
3(a)	$36=\frac{1}{2}\times x\times(x+7)\times\sin 30^{\circ}\Rightarrow 36=\frac{x(x+7)}{4}\Rightarrow x^2+7x-144=0$ $\left[\text{Accept also e.g., } x^2+7x=144\right]$ $(x-9)(x+16)=0\Rightarrow x=9,(-16)\,(\text{cm})$ *	M1 M1A1 cso [3]
(b)	$BC=\sqrt{9^2+16^2-2\times9\times16\cos30^{\circ}}=9.3586...\approx 9.36\,(\text{cm})$	M1A1 [2]
(c)(i) **(ii)**	$\frac{\sin ABC}{16}=\frac{\sin 30^{\circ}}{9.3586}\Rightarrow \angle ABC=180^{\circ}-58.7408...\approx 121.3^{\circ}$ $\angle BCA=180^{\circ}-30^{\circ}-121.259^{\circ}=28.740...\approx 28.7^{\circ}$	M1A1 B1ft [3]
		Total 8 marks

Part	Mark	Notes
(a)	M1	Applies $\frac{1}{2}ab\sin C$ **correctly** with the **given** lengths to form a 3TQ in terms of x
	M1	Solves their 3TQ by any correct method to find the value of x **This is a show question – this step MUST be seen**
	A1 cso	For $x=9$ Sight of $x=-16$ not rejected is A0
(b)	M1	Applies cosine rule correctly to find the length BC
	A1	For the correct length awrt 9.36 (cm)
	ALT	
	M1	Finds the height of the triangle from vertex B [4.5 cm] **and** applies trigonometry or Pythagoras theorem to find the length of the point from the base of the perpendicular to C $AX^2=9^2-4.5^2=\frac{243}{4}\Rightarrow XC=16-\sqrt{\frac{243}{4}}$ $BC=\sqrt{4.5^2+\left(16-\sqrt{\frac{243}{4}}\right)^2}=....$
	A1	For the correct length awrt 9.36 (cm)

(c)	M1	Uses any appropriate trigonometry to find angle *ABC* Award this mark for awrt 58.7… ° or an acute angle if their *AC* is incorrect.
	A1	For $\angle ABC = 180^\circ - 58.7408... \approx 121.3^\circ$ Accept awrt 121.3° or 121.2°
	B1ft	For awrt 28.7° or 28.8° Allow a ft here from their angle *BCA* **NB: this is an A mark in Epen**
	ALT 1 Uses trigonometry again for the final B mark	
	B1ft	**Sine rule** $\frac{\sin ACB}{9} = \frac{\sin 30^\circ}{'9.3586'} \Rightarrow \left[\angle BCA \approx 28.7^\circ\right]$ or $\frac{\sin ACB}{9} = \frac{\sin '121.3^\circ '}{16} \Rightarrow \left[\angle BCA \approx 28.7^\circ\right]$ **cosine rule** $\angle ACB = \cos^{-1}\left(\frac{9.36^2 + 16^2 - 9^2}{2 \times 9.36 \times 16}\right) = \left[28.7^\circ\right]$ For awrt 28.7° allow 28.8°
	ALT 2 Uses the area of a triangle	
	M1	For using the given area of the triangle with their *BC* For example; $36 = \frac{1}{2} \times 9 \times '9.36' \times \sin ABC \Rightarrow \sin ABC = 0.85470... = \left(58.726...^\circ\right)$
	A1	$180^\circ - 58.726...^\circ \approx 121.3^\circ$ or 121.2°
	B1ft	As above or restarts using the area of the triangle. $36 = \frac{1}{2} \times 16 \times '9.36' \times \sin ACB \Rightarrow \left(\angle ACB = 28.7^\circ\right)$

Question	Scheme	Marks
4a)	$\theta = \dfrac{21-r}{r}$	B1
	$A = \dfrac{r^2}{2}\theta \Rightarrow A = \dfrac{r^2}{2}\left(\dfrac{21-r}{r}\right) \Rightarrow A = \dfrac{r}{2}(21-r)$ *	M1A1 cso [3]
	ALT	
	OR $\theta r = 21-r$ or $l = 21-r$	[B1
	$A = \dfrac{r}{2}r\theta$ or $A = \dfrac{1}{2}rl \Rightarrow A = \dfrac{r}{2}(21-r)$ *	M1A1]
(b)	$27 \leqslant \dfrac{r}{2}(21-r) \Rightarrow 54 \leqslant 21r - r^2 \Rightarrow r^2 - 21r + 54 \leqslant 0$	B1
	Finds c.v's	
	$r^2 - 21r + 54 = 0 \Rightarrow (r-18)(r-3) = 0 \Rightarrow r = 18, 3$	M1
	$3 \leqslant r \leqslant 18$	M1A1 [4]
(c)	$\theta = \dfrac{21-r}{r} \Rightarrow \theta = \dfrac{21-18}{18} = \dfrac{1}{6} \quad \theta = \dfrac{21-3}{3} = 6$	B1ft
	$\Rightarrow \dfrac{1}{6} \leqslant \theta \leqslant 6$	B1 [2]
		Total 9 marks

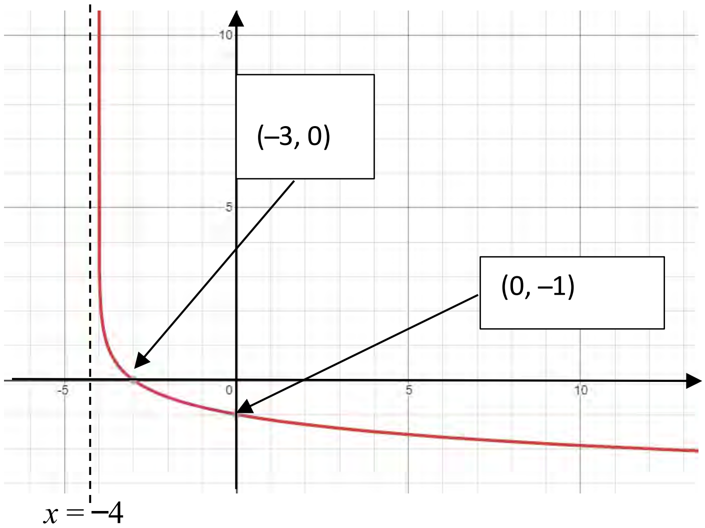
Part	Mark	Notes
(a)	B1	States or uses $\theta = \frac{21-r}{r}$
	M1	Uses the correct formula for the area of a sector and substitutes their expression for θ
	A1 cso	For the correct expression for A exactly as written with no errors. That is $A = \frac{r}{2}(21-r)$ including A =.. However, if they have a string of working, then allow A = at the top For example ., A = ... =.... $= \ldots\ldots\ldots\ldots = \frac{r}{2}(21-r)$ is acceptable
	ALT	
	B1	States $l = 21 - r$
	M1	Uses the alternative formula $A = \frac{1}{2}rl$ and substitutes in their l
	A1 cso	For the correct expression for A exactly as written with no errors.
(b)	B1	Forms a correct inequality **and** a correct 3TQ (accept in any form) using the given expression for A and the value of 27
	M1	Solves their 3TQ by any method to obtain two values for r If they use a calculator to solve an incorrect 3TQ, award only if a method is seen. If the 3TQ is correct, and they use a calculator, award for correct roots seen.
	M1	Forms an inequality for the **inside** region using their values of r Allow any critical values which were obtained from their quadratic equation. Accept any acceptable form that defines a continuous **inside** region For example: $3 \leqslant r$ and $r \leqslant 18$ or $3 \leqslant r \cap r \leqslant 18$ Accept even $3 \leqslant r$ or $r \leqslant 18$
	A1	For the correct inequality only in this form. $3 \leqslant r \leqslant 18$
(c)	B1ft	For both $\theta = \frac{1}{6}$ and $\theta = 6$ You must follow through their values for r for this mark.
	B1	For the correct inequality only in this form.. $\frac{1}{6} \leqslant \theta \leqslant 6$

Question	Scheme	Marks
5(a)	**Mark both parts together**	
	$4k+1=a+(6-1)d \Rightarrow [\ 4k+1=a+(6-1)d]$	M1
	$36k+1=\frac{10}{2}(2a+9d) \Rightarrow [36k+1=10a+45d]$	M1
	Solves the simultaneous equations by any method:	M1
(i)	$d=\frac{4k+9}{5}$	A1
(ii)	$a=-8$*	A1 cso [5]
(b)	$7=-8+3d \Rightarrow 7=-8+3\left(\frac{4k+9}{5}\right) \Rightarrow 25=4k+9 \Rightarrow k=4$ *	M1A1 cso [2]
(c)	$\left[d=\frac{4\times4+9}{5}=5\right.$ you may see this in part (c)$\left.\right]$	
	$\frac{n}{2}(2\times-8+(n-1)\times5)=5[-8+(n+10-1)5]+105$	M1
	$\Rightarrow -21n+5n^2=370+50n+210 \Rightarrow 5n^2-71n-580=0$	dM1
	$\Rightarrow (5n+29)(n-20)=0 \Rightarrow n=20$	M1A1 [4]
		Total 11 marks

Part	Mark	Notes
(a)	**Mark parts (i) and (ii) together.** **If they find *a* first score M1M1M1-**	
	M1	For using the correct formula to write down: $4k+1=a+(6-1)d$
	M1	For using the correct formula to write down: $36k+1=\frac{10}{2}(2a+9d)$
(i)	M1	For an explicit, complete method to solve their two simultaneous equations to find a value for d in terms of k, and a value for a Allow **one** processing error. **Note: This is a show question**
(ii)	A1	For the correct expression for d
	A1 cso	For finding $a=-8$ With no errors, this is a given value. Award this if they find only a but not d
(b)	M1	Uses a correct nth term with their expression for d to form a linear equation **and** attempts to solve it.
	A1 cso	For $k=4$ With no errors, this is a given value.
(c)	M1	For setting up the equation as required using the given a and their d No simplification is required for this mark.
	dM1	For forming a 3TQ using the given a and their d Allow one error: $5n^2-71n-K=0$ Or $5n^2-Ln-580=0$ Or $Mn^2-71n-580=0$ Where K, L and M are constants. Note: this mark is dependent on the previous mark being scored.
	M1	For solving their 3TQ using any valid, correct method If they use a calculator and the 3TQ is correct together with the correct final value, award this mark. If the 3TQ is NOT correct, they must show us a correct valid method for solving the 3TQ for the award of this mark.
	A1	For $n = 20$ with no other value.

Question	Scheme	Marks
6(a)	$t=0 \Rightarrow s=\mathrm{e}^{2\times 0}\sin(3\times 0)+2=2$ $t=\frac{\pi}{6} \Rightarrow s=\mathrm{e}^{2\times\frac{\pi}{6}}\sin\left(3\times\frac{\pi}{6}\right)+2=\mathrm{e}^{\frac{\pi}{3}}+2$ $AB=\mathrm{e}^{\frac{\pi}{3}}+2-2=\mathrm{e}^{\frac{\pi}{3}}$ (m)	M1 A1 [2]
(b)	$v=2\mathrm{e}^{2t}\sin 3t+3\mathrm{e}^{2t}\cos 3t$ When $t=\frac{\pi}{3}$ $v=2\mathrm{e}^{\frac{2\pi}{3}}\sin\left(\frac{\pi}{3}\times 3\right)+3\mathrm{e}^{\frac{2\pi}{3}}\cos\left(\frac{\pi}{3}\times 3\right)=-3\mathrm{e}^{\frac{2\pi}{3}}$ (m/s)	M1A1A1 B1ft [4]
		Total 6 marks

Part	Mark	Notes
(a)	M1	For an attempt to find the displacements of A **and** B as simplified values. At least one must be correct and simplified for this mark and there must be an attempt at the other.
	A1	For finding the distance AB
(b)	M1	For an attempt at product rule. • The formula must be correct. i.e., $[v]=k\mathrm{e}^{2t}\times\sin 3t+\mathrm{e}^{2t}\times l\cos 3t$ • There must be an acceptable attempt to differentiate both e^{2t} and $\sin 3t$ $\mathrm{e}^{2t}\to k\mathrm{e}^{2t}$ and $\sin 3t\to l\cos 3t$ $k,l\neq 0$ and integers • The terms must be added.
	A1	One term fully correct.
	A1	Both terms fully correct.
	B1ft	For using $t=\frac{\pi}{3}$ in their v to find a value for v The minimally acceptable expression for v must be a **changed** expression from the given s in terms of e^{2x}, $\sin 3x$ and $\cos 3x$ Note: This is a ft mark. Please check their substitution and the final result. Please isw a value of $-24.36....$ if seen **together** with the exact value.

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>7(a)</td><td>
</td><td>B1
Shape and position

B1
Intersection with x-axis

B1
Intersection with y-axis

B1
Asymptote
[4]</td></tr>
<tr><td>(b)</td><td>$\log_{(x+4)} 256 - \log_4(x+4) = 0 \Rightarrow \dfrac{\log_4 256}{\log_4(x+4)} - \log_4(x+4) = 0$

$\Rightarrow 4 - \left(\log_4(x+4)\right)^2 = 0 \Rightarrow \left(\log_4(x+4)\right)^2 = 4$

$\Rightarrow \log_4(x+4) = \pm 2$

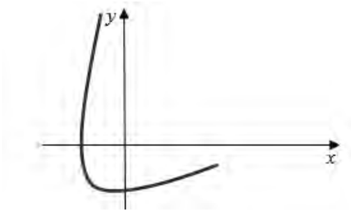
$x + 4 = 4^2 \Rightarrow x = 12$

$x + 4 = 4^{-2} \Rightarrow x = -\dfrac{63}{16}$ or -3.9375</td><td>M1

M1

M1
A1

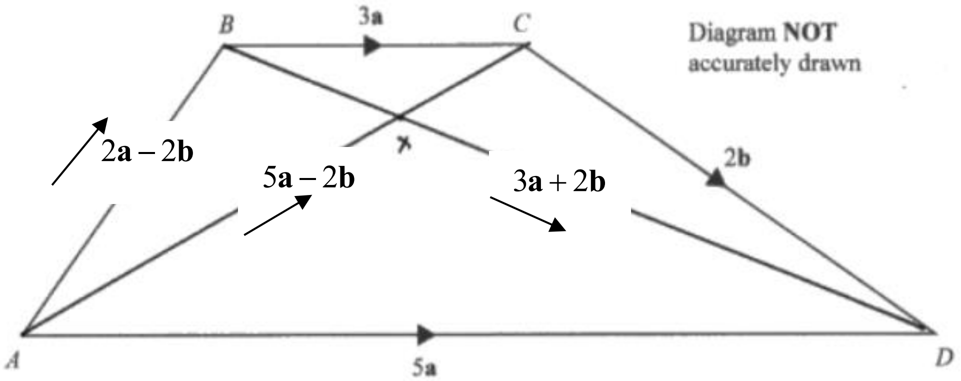
A1
[5]</td></tr>
<tr><td colspan="3" align="right">Total 9 marks</td></tr>
</table>

Part	Mark	Notes
(a)	B1	For the correct shape in the correct position. Ignore intersections and the asymptote for this mark. Award as long as the curve is the correct way around in quadrants 2, 3 and 4. The ends must not turn back in themselves. Do Not accept for example:  If there are two or more lines, and candidates have not made clear which ONE they want marked– award B0
	B1	For a line/curve **passing** through (−3, 0) or accept $x = -3$ Do not accept stopping at the axis.
	B1	For a line/curve passing through (0, −1) or accept $y = -1$ Do not accept stopping at the axis.
	B1	For the correct **equation** only of the asymptote. If they also give a horizontal asymptote then do not isw – it is B0
(b)	**Method 1 - Works in log base 4**	
	M1	For changing the base of the log to base 4
	M1	For forming a quadratic in terms of $(x + 4)$ Accept a substitution for the log
	M1	For taking the square root **and** undoing the log. Accept just the positive root for this mark.
	A1	For either $x = 12$ or $-\frac{63}{16}$
	A1	For both $x = 12$ and $-\frac{63}{16}$
	Method 2 – Works in log base $(x + 4)$	
	M1	For changing the base of the log to base $(x + 4)$ $\log_{(x+4)} 256 - \frac{\log_{(x+4)}(x+4)}{\log_{(x+4)} 4} = 0 \Rightarrow$
	M1	Changes $\log_{(x+4)}(x+4) = 1$ and $\log_{(x+4)} 256 = 4\log_{(x+4)} 4$ And forming a quadratic equation with log base (x + 4) $\left(\log_{(x+4)} 4\right)^2 = \frac{1}{4}$ Accept a substitution for the log
	M1	Takes the square root (accept just the positive root) **and** undoes the log $\log_{(x+4)} 4 = \pm\frac{1}{2} \Rightarrow 4 = (x+4)^{\frac{1}{2}}$ and $4 = (x+4)^{-\frac{1}{2}}$
	A1	For either $x = 12$ or $-\frac{63}{16}$
	A1	For both $x = 12$ and $-\frac{63}{16}$

	Method 3 works in an unspecified base. If it is just log, assume base 10	
	M1	For changing the base $\frac{\log 256}{\log(x+4)} - \frac{\log(x+4)}{\log 4} = 0$
	M1	Forms a QE in terms of $(x + 4)$ $\quad 4(\log 4)^2 = (\log(x+4))^2$ oe
	M1	Takes square root (accept just the positive root) and undoes the log $\pm 2\log 4 = \log(x+4) \Rightarrow \log 4 = \pm\frac{1}{2}\log(x+4)$ $\Rightarrow \log 4 = \log(x+4)^{\pm\frac{1}{2}} \Rightarrow 4 = (x+4)^{\pm\frac{1}{2}}$ $\Rightarrow 16 = x+4, \quad \frac{1}{16} = x+4$
	A1	For either $x = 12$ or $-\frac{63}{16}$
	,A1	For both correct values $x = 12$ and $-\frac{63}{16}$

Question	Scheme	Marks
8(a)	$\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DC} + \overrightarrow{CB} = 5\mathbf{a} - 2\mathbf{b} - 3\mathbf{a} = 2\mathbf{a} - 2\mathbf{b}$	B1 [1]
(b)	$\overrightarrow{BX} = k\,\overrightarrow{BD} = k(3\mathbf{a} + 2\mathbf{b})$	M1
	$\overrightarrow{BX} = \overrightarrow{BA} + \lambda\,\overrightarrow{AC} = -2\mathbf{a} + 2\mathbf{b} + \lambda(5\mathbf{a} - 2\mathbf{b})$	M1
	$\Rightarrow k(3\mathbf{a} + 2\mathbf{b}) = -2\mathbf{a} + 2\mathbf{b} + \lambda(5\mathbf{a} - 2\mathbf{b})$	
	$\Rightarrow 3k\mathbf{a} + 2k\mathbf{b} = (5\lambda - 2)\mathbf{a} + (2 - 2\lambda)\mathbf{b}$	M1
	$\Rightarrow 3k = 5\lambda - 2 \quad 2k = 2 - 2\lambda$	
	$\Rightarrow k = \frac{3}{8} \quad \left[\lambda = \frac{5}{8}\right]$	M1A1 [5]
(c)	$\Delta CXD = \frac{5}{8}\Delta BCD$	M1
	$\Delta BCD = \frac{3}{8} ABCD$	M1
	$\frac{\Delta CXD}{ABCD} = \frac{\frac{5}{8}}{\frac{8}{3}} = \frac{15}{64}$	M1
	Ratio of area of triangle *CXD* : Trapezium *ABCD* = $15:64$	A1 [4]
		Total 10 marks

Useful sketch



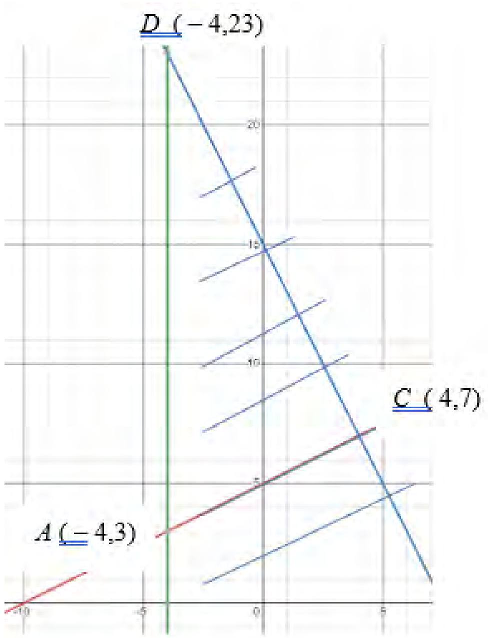
Part	Mark	Notes
(a)	B1	For the correct vector for $\overrightarrow{AB}$
(b)	**Note carefully.** For the first two marks in part (b) you must follow through their working using their $\overrightarrow{AB}$ or $\overrightarrow{AC}$ or $\overrightarrow{BD}$	
	M1	For one correct vector statement for $\overrightarrow{BX}$
	M1	For a second correct and different vector statement for $\overrightarrow{BX}$ For example: $\overrightarrow{BX} = \overrightarrow{BA} + \lambda\overrightarrow{AC} = -2\mathbf{a} + 2\mathbf{b} + \lambda(5\mathbf{a} - 2\mathbf{b})$ OR $\overrightarrow{BX} = \overrightarrow{BC} + \lambda\overrightarrow{AC} = 3\mathbf{a} - \lambda(5\mathbf{a} - 2\mathbf{b})$
	ddM1	For equating coefficients of **a** and **b** and forming two equations in k and another parameter. This must be correct **Dependent on first two M marks**
	dddM1	For solving their simultaneous equations to find k Allow one arithmetical error in processing. **Dependent on all three previous M marks**
	A1	For $k = \frac{3}{8}$
(c)	M1	For using their k to find the ratio of the areas of for example, $CXD : CAD \Rightarrow \frac{CXD}{CAD} = \frac{3}{8}$ or $\frac{CXD}{CAD} = \frac{\frac{1}{2}\times\frac{3}{8}\times 2b\times\sin ACD}{\frac{1}{2}\times\frac{8}{8}\times 2b\times\sin ACD} = \frac{3}{8}$ **OR** $CXD : BCD \Rightarrow \frac{CXD}{BCD} = \frac{5}{8}$ or $\frac{CXD}{CAD} = \frac{\frac{1}{2}\times\frac{5}{8}\times 2b\times\sin BDC}{\frac{1}{2}\times\frac{8}{8}\times 2b\times\sin BDC} = \frac{5}{8}$
	M1	For using the given lengths of BC and AD [3 and 5],to find the ratio of the area of triangle ACD : trapezium $ABCD$ $\frac{CAD}{ABCD} = \frac{\frac{1}{2}\times h\times 5}{\frac{1}{2}\times h\times(5+3)} = \frac{5}{8}$ **OR** $\frac{BCD}{ABCD} = \frac{\frac{1}{2}\times h\times 3}{\frac{1}{2}\times h\times(5+3)} = \frac{3}{8}$
	Note carefully. You must ft their ratios above and check to see if they are being combined correctly for the final M mark here.	
	M1	Combines the areas above to obtain: $\frac{CXD}{CAD}\times\frac{CAD}{ABCD} = \frac{3}{8}\times\frac{5}{8} = \left(\frac{15}{64}\right)$ **OR** $\frac{CXD}{BCD}\times\frac{BCD}{ABCD} = \frac{5}{8}\times\frac{3}{8} = \left(\frac{15}{64}\right)$
	A1	For the correct ratio $15:64$

Question	Scheme	Marks
9(a)	$\dfrac{y-8}{8-3}=\dfrac{x-6}{6--4}\Rightarrow\dfrac{y-8}{5}=\dfrac{x-6}{10}\Rightarrow\left[y-8=\dfrac{1}{2}(x-6)\right]$	M1A1 [2]
(b)	$C=\left(\dfrac{4\times6+1\times-4}{4+1},\dfrac{4\times8+1\times3}{4+1}\right)=(4,7)$	B1,B1 [2]
(c)	$-2=\dfrac{q-7}{p-4}\Rightarrow q=15-2p$ $8\sqrt{5}=\sqrt{(4-p)^2+(7-q)^2}$ $\Rightarrow 320=(4-p)^2+(2p-8)^2\Rightarrow p^2-8p-48=0$ $\Rightarrow(p+4)(p-12)=0$ $\Rightarrow p=-4\quad q=15-2\times-4=23\Rightarrow D=(-4,23)$	M1 M1 M1 M1 A1A1 [6]
(d)	Area of triangle ACD = $\dfrac{1}{2}\times(23-3)\times(4--4)=80$ [square units] **ALT 1** $\dfrac{1}{2}\begin{bmatrix}-4 & 4 & -4 & -4\\ 3 & 7 & 23 & 3\end{bmatrix}=\dfrac{1}{2}\left[(-28+92-12)-(12-28-92)\right]=80$ **ALT 2** $A=\dfrac{1}{2}\times AC\times CD=\dfrac{1}{2}\times4\sqrt{5}\times8\sqrt{5}=80$	M1A1 [2] [M1A1] [M1A1]
	Total 12 marks	

Part	Mark	Notes
(a)	M1	For a **complete** method to find an equation of the line segment AB They must either use the formula shown Award M1A1 for $\dfrac{y-8}{5}=\dfrac{x-6}{10}$ (the denominators must be processed). OR They must find the gradient using a correct method and then use the formula $y-y_1=m(x-x_1)\Rightarrow y-8='\dfrac{1}{2}'(x-6)$ or using $(-4,3)$ OR They must find the gradient using a correct method and then using $y=mx+c$ they must find c and **put an equation together** to score this mark. $y='\dfrac{1}{2}'x+'5'$ Allow for example $k=\dfrac{1}{2}x+5$ for this mark (M1) only.
	A1	For a correct equation in any form.[See above]

(b)	B1	For either correct coordinate of C (4, 7) **This is an M mark in Epen**
	B1	For both correct coordinates of C (4, 7) **This is an M mark in Epen**
(c)	**Note:** You must ft their coords of C in part (c)	
	M1	For using the perpendicular gradient of their equation in (a) to set up a correct equation in terms of p and q **OR** For finding the equation of the line CD using their point C (4, 7) The method must be complete for this mark. $\frac{y-7}{7-23}=\frac{x-4}{4--4} \Rightarrow y=-2x+15$
	M1	For using the given length of CD to set up a second equation using Pythagoras theorem in terms of p and q $8\sqrt{5}=\sqrt{(4-p)^2+(7-q)^2}$ oe
	M1	For combining their two equations in p and q to obtain a 3TQ
	M1	For solving their 3TQ (provided it is a 3TQ) by any acceptable method. If the 3TQ is incorrect, there must be evidence of a method to score this mark. If they use their calculator on a correct 3TQ and obtain the correct values, score M1.
	A1	For the correct value of p or q
	A1	For the correct values of both p and q
(d)	M1	For any correct method to find the area of triangle ACD, using their coordinates. If they use the determinant method, check their work carefully.
	A1	For the correct area of 80 [square units].

Useful sketch



Question	Scheme	Marks
10(a)	$\alpha+\beta=-\frac{k}{2} \qquad \alpha\beta=2$	B1
	$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta, \quad \alpha^2-\beta^2=(\alpha+\beta)(\alpha-\beta)$	B1B1
	$\Rightarrow \alpha^2-\beta^2=(\alpha+\beta)\sqrt{(\alpha+\beta)^2-4\alpha\beta}$	
	$\Rightarrow \alpha^2-\beta^2=\left(-\frac{k}{2}\right)\sqrt{\left(-\frac{k}{2}\right)^2-4\times 2}=\frac{7\sqrt{17}}{4}$	M1
	$\Rightarrow \frac{k^4}{4}-8k^2-\frac{833}{4}=0 \Rightarrow k^4-32k^2-833=0$ or any equivalent	M1A1
	$\Rightarrow (k^2-49)(k^2+17)=0 \Rightarrow k=-7$ *	M1A1 cso [8]
(b)	Product: $\left[(\alpha-\beta)(\alpha+\beta)=\alpha^2-\beta^2=\frac{7\sqrt{17}}{4}\right]$ from (a)	B1
	Sum: $(\alpha-\beta)+(\alpha+\beta)=\frac{\sqrt{17}}{2}+\frac{7}{2}=\left(\frac{\sqrt{17}+7}{2}\right)$	M1
	$x^2-\left(\frac{\sqrt{17}+7}{2}\right)x+\frac{7\sqrt{17}}{4}=0 \Rightarrow 4x^2-2\left(\sqrt{17}+7\right)x+7\sqrt{17}=0$	M1A1 [4]
		Total 12 marks

There is an alternative solution using the roots of the equation.

Roots are $\frac{-k\pm\sqrt{k^2-4\times 2\times 4}}{2\times 2}$

$$\Rightarrow \left(\frac{-k+\sqrt{k^2-32}}{4}\right)^2-\left(\frac{-k-\sqrt{k^2-32}}{4}\right)^2=\frac{7\sqrt{17}}{4}$$

$$\Rightarrow \left(-k+\sqrt{k^2-32}\right)^2-\left(-k-\sqrt{k^2-32}\right)^2=28\sqrt{17}$$

$$\Rightarrow k^2-2k\sqrt{k^2-32}+k^2-32-k^2-2k\sqrt{k^2-32}-k^2+32=28\sqrt{17}$$

$$\Rightarrow -4k\sqrt{k^2-32}=28\sqrt{17} \Rightarrow k\sqrt{k^2-32}=7\sqrt{17}$$

$$\Rightarrow k^2\left(k^2-32\right)=833 \Rightarrow k^4-32k^2-833=0$$

$$\Rightarrow \left(k^2-49\right)\left(k^2+17\right)=0 \Rightarrow k=-7 \text{ *}$$

Part	Mark	Notes
(a)	B1	For the correct sum in terms of k and the correct product. Look out for these embedded in their work.
	B1	For the correct algebra on $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$ Substitution not required for this mark, although it can be implied from correct substitution without the actual algebra seen.
	B1	For the correct algebra on $\alpha^2-\beta^2$ Award also for $(\alpha+\beta)(\alpha-\beta)=\frac{7\sqrt{17}}{4}$ Substitution not required for this mark, although it can be implied from correct substitution without the actual algebra seen.
	M1	For substituting in the values of their sum and product into their $\alpha^2-\beta^2$ This can be implied from sight of $\left(-\frac{k}{2}\right)\sqrt{\left(-\frac{k}{2}\right)^2-4\times 2}=\frac{7\sqrt{17}}{4}$
	M1	For forming a 3TQ in terms of k^2 in any form
	A1	For the correct 3TQ Accept it in any form as long as it is 3 terms. For example, accept $\frac{k^4}{16}-2k^2-\frac{833}{16}=0$ etc
	M1	For any acceptable attempt seen to solve their 3TQ [see General Guidance] If there is no working, just $k=-7$ following a 3TQ then award M0 However, accept evidence of $k^2=49$ $\left[\text{with } k^2=-17\right]$
	A1 cso	For the correct value of k * **Note: This is a given value**
(b)	B1	For a value for the product using their results from (a) or just writes it down.
	M1	For a correct method to find the value of the sum using their values for $(\alpha-\beta)$ and $(\alpha+\beta)$
	M1	For forming a 3TQ with their sum and product. = 0 is not required for the award of this mark
	A1	For a correct equation simplified or unsimplified. For example, accept, $x^2-\left(\frac{\sqrt{17}+7}{2}\right)x+\frac{7\sqrt{17}}{4}=0$ as well as $4x^2-2\left(\sqrt{17}+7\right)x+7\sqrt{17}=0$ o.e.

Question	Scheme	Marks
11(a)	$\mathrm{f}(\theta)=(2\cos\theta-\sin\theta)(2\sin\theta+\cos\theta)$ $=4\sin\theta\cos\theta+2\cos^2\theta-2\sin^2\theta-\sin\theta\cos\theta$ $=3\sin\theta\cos\theta+2(\cos^2\theta-\sin^2\theta)\Rightarrow\frac{3}{2}\sin 2\theta,+2\cos 2\theta$ $=\frac{3}{2}\sin 2\theta+2\cos 2\theta$ *	M1 M1A1 cso [3]
(b)	$\frac{3}{2}\sin 2\theta+2\cos 2\theta+2=0\Rightarrow 3\sin 2\theta+4\cos 2\theta+4=0$ $\Rightarrow 3\sin 2\theta+4(\cos^2\theta-\sin^2\theta)+4(\sin^2\theta+\cos^2\theta)=0$ $\Rightarrow 6\sin\theta\cos\theta+8\cos^2\theta=0$ $\Rightarrow \cos\theta(6\sin\theta+8\cos\theta)=0$ $\Rightarrow \cos\theta=0\Rightarrow\theta=\frac{\pi}{2}$ * $\left[\Rightarrow \tan\theta=-\frac{4}{3}\right]$	M1M1 M1 M1 A1 cso [5]
(c)	Area $=\int_0^{\frac{\pi}{2}}\left(\frac{3}{2}\sin 2\theta+2\cos 2\theta+2\right)\mathrm{d}\theta=\left[-\frac{3}{4}\cos 2\theta+\frac{2\sin 2\theta}{2}+2\theta\right]_0^{\frac{\pi}{2}}$ $=\left(-\frac{3}{4}\cos 2\left(\frac{\pi}{2}\right)+\frac{2\sin 2\left(\frac{\pi}{2}\right)}{2}+2\left(\frac{\pi}{2}\right)\right)-\left(-\frac{3}{4}\cos 0+\frac{2\sin 0}{2}+2\times 0\right)$ $=\left(\frac{3}{4}+\pi\right)-\left(-\frac{3}{4}\right)=\frac{3}{2}+\pi$	M1 M1 A1 [3]
	Total 11 marks	

Part	Mark	Notes
(a)	M1	For multiplying out the brackets **correctly** and simplifying to $k\sin\theta\cos\theta,\quad l(\cos^2\theta-\sin^2\theta)$ where k and l are integers**.** Condone invisible brackets if the working is clear.
	M1	For using the identity for $\cos 2\theta$ on their $2(\cos^2\theta-\sin^2\theta)$ **OR** For using the identity for $\sin 2\theta$ on their $3\sin\theta\cos\theta$
	A1 cso	For the correct identity as shown with no errors. You must check every line of working carefully – this is a given answer.
		Solutions based on RHS = LHS are acceptable. Apply the above marks – if you are not sure, please send to REVIEW

(b)	M1	For correctly using the identity $\cos 2\theta = \cos^2\theta - \sin^2\theta$
	M1	For correctly using the identity $\sin^2\theta + \cos^2\theta = 1$
	M1	For correctly using the identity $\sin 2\theta = 2\sin\theta\cos\theta$
	M1	For factorising the resulting expression. You must see this step
	A1 cso	For $\cos\theta = 0 \Rightarrow \theta = \dfrac{\pi}{2}$
	ALT	
	M1	For correctly using $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\frac{3}{2}\sin 2\theta + 2\cos 2\theta + 2 = \frac{3}{2}\sin 2\theta + 2\left(\cos^2\theta - \sin^2\theta\right) = 0$
	M1	For correctly using the identity $\sin^2\theta + \cos^2\theta = 1$ $\frac{3}{2}\sin 2\theta + 2\cos 2\theta + 2 = \frac{3}{2}\sin 2\theta + 2\left(\cos^2\theta - 1 + \cos^2\theta\right) + 2 = 0$ $\Rightarrow \left[\frac{3}{2}\sin 2\theta + 4\cos^2\theta = 0\right]$
	M1	For correctly using the identity $\sin 2\theta = 2\sin\theta\cos\theta$ $\frac{3}{2}\sin 2\theta + 4\cos^2\theta = 3\sin\theta\cos\theta + 4\cos^2\theta = 0$
	M1	For factorising the resulting expression. You must see this step $3\sin\theta\cos\theta + 4\cos^2\theta = \cos\theta\left(3\sin\theta + 4\cos\theta\right) = 0$
	A1	$\cos\theta = 0 \Rightarrow \theta = \dfrac{\pi}{2}$ *
(c)	M1	For integrating the given expression. Minimally acceptable integration as shown below: $\frac{3}{2}\sin 2\theta \Rightarrow \pm\frac{3}{4}\cos 2\theta$ $2\theta\cos 2\theta \Rightarrow \pm\left(\frac{2}{2}\right)\sin 2\theta$ $2 \Rightarrow 2\theta$
	M1	For substituting **BOTH** of the given limits the correct way around into **their changed expression** and subtracting.. There must be a minimum of two terms to substitute limits into. This must be explicitly seen
	A1	For the correct exact area as shown. [Approximate area = 4.641….]

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